Uncertainty quantification of segmentation and anisotropic local geometric prior

Valeriy Vishnevskiy,
supervised by Grégory Paul, Ivo Sbalzarini

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Active contour, snake models

- Параметризуем контур сегмента некоторой кривой, минимизируем энергию.
- $$E(C) = \alpha \int_0^1 E_{int}\left(C\left(p\right)\right) dp + \beta \int_0^1 E_{img}\left(C\left(p\right)\right) dp + \gamma \int_0^1 E_{ext}\left(C\left(p\right)\right) dp.$$
- $E_{int} = \alpha |C_p(p)|^2 + \beta |C_{pp}(p)|$, гладкость, регуляризация.
- $E_{img} = w_1 I(C(p)) - w_2 |\nabla I(C(p))|$, выкладывает контур по желаемым свойствам.
- E_{ext} используется для задания других ограничений.
- balloon models, region snakes, etc...

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Level Sets

- Контур $C(t)$ задается неявно: нулевым уровнем функции $\phi(x, y, t)$.

$$C(p, t) = \{(x, y) \colon \phi(x, y, t) = 0\}, \ \phi(C(t), t) = 0.$$

- Модель изменения контура: $\frac{dC}{dt} = F\vec{n}$.
- Тогда получаем следующее уравнение:

$$\phi_t = -F|\nabla\phi|.$$

Geodesic active contours, edge-driven case

- Связь со snake model:

$$E\left[C(p)\right] = \alpha \int_0^1 \left|\frac{\partial C}{\partial p}(p)\right|^2 dp + \beta \int_0^1 g(C(p))dp$$

- Strong image gradients $g(x,y) = \frac{1}{1+|\nabla G_s * I(x,y)|^2}$.
-

$$\phi_t = |\nabla \phi| div\left(g(p)\frac{\nabla \phi}{|\nabla \phi|}\right)$$

The region-driven case. Mumford-Shah framework

- Кусочно-константая(гладкая) модель изображения:

$$E(C,u) = \alpha \iint_{\Omega} (I-u)^2 d\omega + \beta|C| + \gamma \iint_{\Omega - C} \|\nabla u\|^2 d\omega.$$

Segmentation

Using model-based image segmentation

$$\left(\hat{\Gamma}, \hat{\beta}\right) = \arg\inf_{\Gamma,\beta} \mathcal{E}_d(\Gamma, \beta) + \lambda\, \mathcal{E}_r(\Gamma)$$

Iteratively repeat

- Geometric reconstruction for Γ (alternating split Bregman solver and convex relaxation);
- Photometric reconstruction for β (MLE in exponential family, Fisher scoring algorithm);

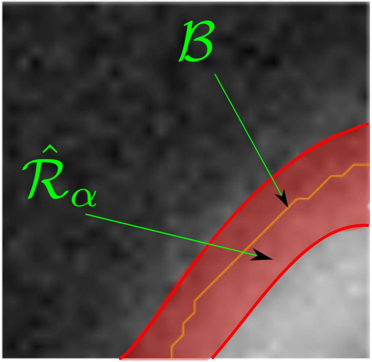
Problem

We want to:

- identify regions, where we are less confident about segmentation,
- estimate confidence intervals for segmentation boundary.

For fixed α $(= 0, 05)$ find $\hat{\mathcal{R}}_\alpha$ such that

$$P\left\{\forall x \in \mathcal{B}, x \in \hat{\mathcal{R}}_\alpha\right\} = 1 - \alpha.$$



Segmentation

Problems:

- alternating process,
- no accurate upper bound for geometric solver,
- only lower bound for variance of MLE,
- we are working with pixels, not the boundary.

We have some idea about the uncertainty in $\hat{\beta}$.

What we can do?

- MLE is asymptotically normal(under some conditions).
- Estimate variance of MLE using Fisher information matrix $\mathcal{I}$.

$$var(\hat{\theta}) \geq \mathcal{I}^{-1}(\hat{\theta})$$

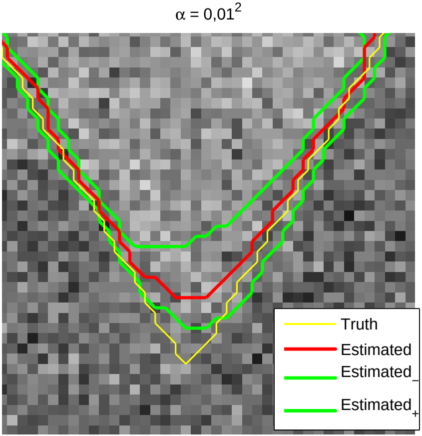
- Since we are working with statistics over pixels in the regions, not on the boundaries, it is reasonable to use $\hat{\alpha} \sim \alpha^2$ in 2D case.

Use normal approximation and propagate error of Region Statistics Solver at each step through the whole process.

$$\beta_+ = \beta_{MLE} + \Phi_{\hat{\alpha}}\sigma \;\longrightarrow\; \text{Geometric solver: } \hat{\Gamma}(\beta_+) \;\longrightarrow\; \mathcal{R}_+$$

$$\beta_- = \beta_{MLE} - \Phi_{\hat{\alpha}}\sigma \;\longrightarrow\; \text{Geometric solver: } \hat{\Gamma}(\beta_-) \;\longrightarrow\; \mathcal{R}_-$$

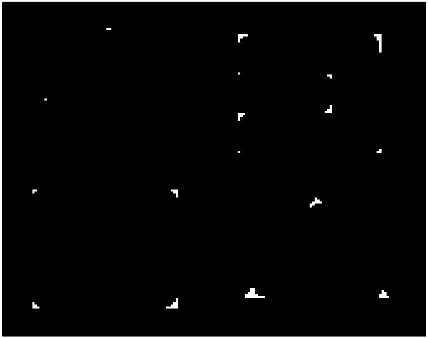
Examples



Geometric solver is biased due to the sharp corners.

Examples

Pixels of objects, that are not in the confidence interval



$$\text{Mean}(500\ \text{trials})\ \frac{Npixels_{\textbf{outside CI}}}{Npixels_{\textbf{in objects}}} = 0.0118.$$

$$SNR = 1.5 \quad PSF_\sigma = 2 \quad PSF_{width} = 9 \quad \alpha = 0.0001$$

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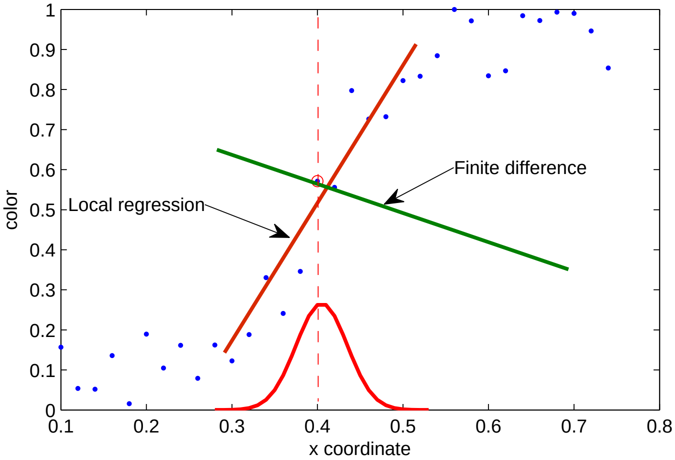
Main concept

Estimate gradients, using linear regression.



Main concept

- Taking window too small leads to high sensitivity,
- too wide window adds bias.

2D images

- At each pixel take weighted neighborhood of size $M \times M$.
- Robustly fit 2D plane with IRLS.

Parameters:

- neighborhood size,
- weight function,
- robustness coefficient,
- “sharpness” of gradients.

Too much smoothing can blur sharp edges.

Example

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Extending $\mathcal{E}_r$ to include local anisotropic priors

- Length prior

$$\mathcal{E}_r(\Gamma) = \int_\Gamma ds = TV(1_\Omega) \text{``}=\text{''} \int_{\Omega_I} \|\nabla 1_\Omega\|_2 dx.$$

- Geodesic active contour prior

$$\mathcal{E}_r(\Gamma) = \int_\Gamma w_b(s) ds = TV_{w_b}(1_\Omega) \text{``}=\text{''} \int_{\Omega_I} w_b(x) \|\nabla 1_\Omega\|_2 dx.$$

  See Bresson et al. 2007

- Finsler active contours

$$\mathcal{E}_r(\Gamma) = \int_\Gamma w_b(x, \vec{n}) ds \text{``}=\text{''} ??$$

Defining the form

Nemitz et al.(2007), Casells (2009).

$$\mathcal{E}_r(\Gamma) = \int_{\Omega_I} \phi(x, \nabla 1_\Omega, \vec{n}) dx, \qquad \phi = \sup_{q \in W_\phi} \langle \nabla M, q \rangle$$

Ellipsoid Wulff shapes

$$W_\phi = \left\{ x \in \mathbb{R}^d \mid \|x\|_{C_{x_q}} := \|C_{x_q} x\|_2 \leq 1 \right\}$$

Generalized co-area formula holds

$$\mathcal{E}_r(\Gamma) \text{``}=\text{''} \int_{\Omega_I} \left\| C_{x_q}^{-1} \nabla 1_\Omega \right\|_2 dx$$

Exact convex relaxation possible by thresholding. Olson, Byrod et al. (ICCV 2009).

Explaining well known priors

- Length prior

$$W_\phi(x_q) = \left\{x \in \mathbb{R}^d \mid \|x\|_2 \leq 1\right\}$$

— independent of x_q.

- Geodesic active contour prior

$$W_\phi(x_q) = \left\{x \in \mathbb{R}^d \mid \|x\|_2 \leq g(x_q)\right\}$$

— smoothing varying in space. Edge indicator controls volume of the Wulff shape.

Convex relaxation:

$$1_\Omega(x) \in \{0,1\} \quad \longrightarrow \quad M(x) \in [0,1].$$

On the relaxed problem use alternating split Bregman:

$$M^{k+1} = \arg\min_M \left\| b_1^k + K[M] - w_1^k \right\|_2^2 + \left\| b_2^k + \nabla M - w_3^k \right\|_2^2 + \left\| b_3^k + M - w_3^k \right\|_2^2$$

$$w_1^{k+1} = \arg\min_{w_1} \langle 1, E(w_1) \rangle + \frac{1}{2\gamma} \left\| b_1^k + K\left[M^{k+1}\right] - w_1 \right\|_2^2$$

$$\boxed{w_2^{k+1} = \arg\min_{w_2} \ \lambda\phi(w_2) + \frac{1}{2\gamma} \left\| b_2^k + \nabla M^{k+1} - w_2 \right\|_2^2}$$

$$w_3^{k+1} = \arg\min_{w_3} \ \iota_{[0,1]}(w_3) + \frac{1}{2\gamma} \left\| b_3^k + M^{k+1} - w_3 \right\|_2^2$$

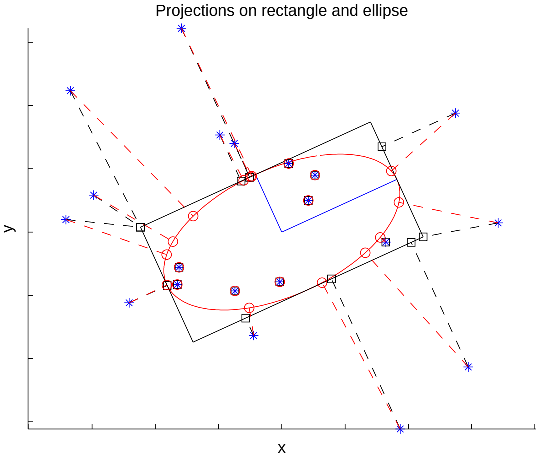
Update $b_1^{k+1}, b_2^{k+1}, b_3^{k+1}$

Solve $\forall x \in \Omega_I$:

$$\begin{aligned}\arg\min_{w \in \mathbb{R}^d} \lambda\gamma\phi(w_2(x)) + \frac{1}{2}\left\|b_2^k(x) + \nabla M^{k+1}(x) - w_2(x)\right\|_2^2 &= \\ = prox_{\lambda\gamma, W_\phi}\left(b_2^k(x) + \nabla M^{k+1}(x)\right) &= \\ = b_2^k(x) + \nabla M^{k+1}(x) - \lambda\gamma \boxed{\Pi_{W_\phi}\left(\frac{b^k(x) + \nabla M^{k+1}(x)}{\lambda\gamma}\right)}&.\end{aligned}$$

Main computational task is Euclidian projection on the Wulff shape.

No closed form solution for projecting point on ellipse.



Using iterative method(2-6 iterations).
David Eberly "Distance from a Point to an Ellipse in 2D"

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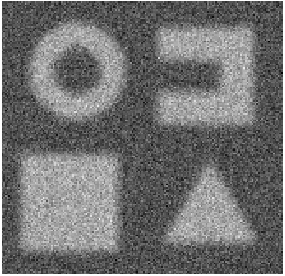
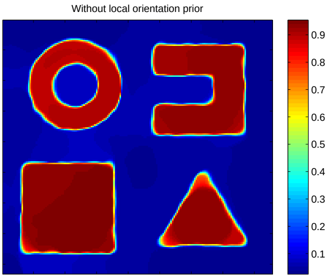
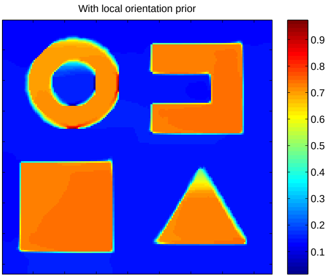
Proof of concept



$$SNR = 4 \quad PSF_\sigma = 3 \quad PSF_{width} = 9$$

Proof of concept

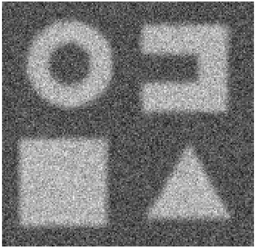




Membership mask. Orientations estimated using unnoised image.

$$SNR = 4 \quad PSF_{\sigma} = 3 \quad PSF_{width} = 9$$

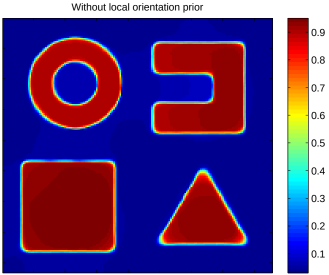
Enhancing corners



$$SNR = 4.5 \quad PSF_\sigma = 2 \quad PSF_{width} = 9$$

Enhancing corners

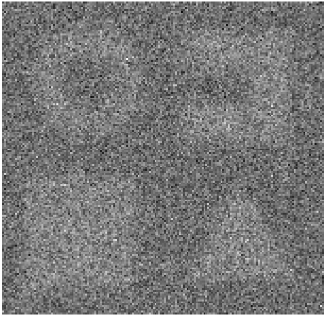
Better corner detection.



Avaraged membership mask.

$$SNR = 4.5 \quad PSF_{\sigma} = 2 \quad PSF_{width} = 9$$

Segmenting extremely noisy images

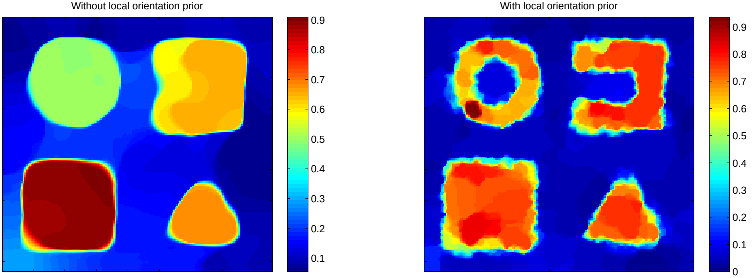


Initial image

$$SNR = 0.8 \quad PSF_\sigma = 3 \quad PSF_{width} = 9$$

Segmenting extremely noisy images

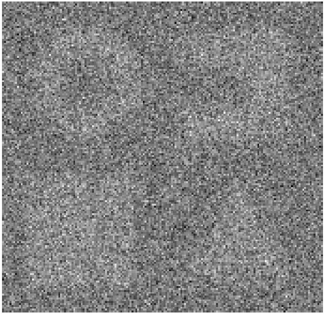
Orientation prior makes it possible to segment noisy images.



Membership mask.

$$SNR = 0.8 \quad PSF_{\sigma} = 3 \quad PSF_{width} = 9$$

Segmenting extremely noisy images

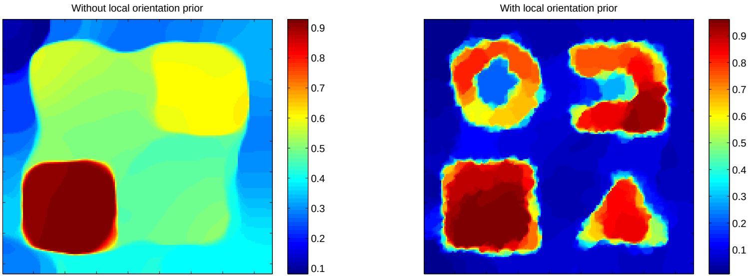


Initial image

$$SNR = 0.6 \quad PSF_{\sigma} = 3 \quad PSF_{width} = 9$$

Segmenting extremely noisy images

Orientation prior helps to reveal the structure of noisy images.



Membership mask.

$$SNR = 0.6 \quad PSF_\sigma = 3 \quad PSF_{width} = 9$$

What had been done.

- Implemented trivial approach for estimating confidence intervals of the segmentation.
- Implemented algorithm for local orientation estimation.
- Improved segmentation of sharp corners.
- Local geometric prior helps to segment noisy($SNR < 1$) images.

What could be improved.

- Estimate orientations more robustly.
- Use orientation priors only near sharp corners.