# Deep <br> Reinforcement Learning with Memory 

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RL basics

## Markov Decision Process

Environment

## Markov Decision Process



## Markov Decision Process

## Agent



## Markov Decision Process



Agent


## Markov Decision Process



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## Markov Decision Process

```
state dynamics
\(s_{t+1} \sim p\left(s_{t+1} \mid s_{t}\right)\)
```



## Markov Decision Process

```
state dynamics
st+1}~~p(\mp@subsup{s}{t+1}{}|\mp@subsup{s}{t}{}
```

    state \(s_{t+1}\)
    


Agent

action $a_{t+1}$
reward structure $r_{t}=r\left(s_{t}, a_{t}, s_{t+1}\right)$


## Markov Decision Process

- Trajectory $s_{0}, a_{0}, s_{1}, r_{0}, a_{1}, s_{2}, r_{1}, \ldots$
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- Action-state function: $Q^{\pi}\left(s_{t}, a_{t}\right)=\mathbb{E}_{s_{t: T}, a_{t+1: T}}\left[\sum_{l=0}^{T} \gamma^{l} r_{t+l}\right]$
- $V^{\pi}\left(s_{t}\right)=\mathbb{E} a_{t} Q^{\pi}\left(s_{t}, a_{t}\right)$


## Bellman equation and Value iteration

- $V^{\pi}\left(s_{t}\right)=r_{t}+\gamma \mathbb{E}_{t+1}\left[V^{\pi}\left(s_{t+1}\right)\right]$
- $Q^{\pi}\left(s_{t}, a_{t}\right)=r_{t}+\gamma \mathbb{E}_{s_{t+1}, a_{t+1}}\left[Q^{\pi}\left(s_{t+1}, a_{t+1}\right)\right]$


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2. Evaluate it to obtain $Q^{\pi}$
3. Improve $\pi$ to $\pi^{\prime}: \pi^{\prime}(a \mid s)=\arg \max Q^{\pi}(s, a)$
4. Repeat until convergence

$$
\min _{Q} \mathbb{E}\left\|Q\left(s_{t}, a_{t}\right)-r_{t}-\gamma Q\left(s_{t+1}, a_{t+1}\right)\right\|^{2}
$$

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- Policy gradient can be written as follows:

$$
\nabla_{\theta} R^{\pi}=\mathbb{E}_{s_{0: T}, a_{0: T}}\left[\sum_{t=0}^{T} \gamma^{t} \sum_{l=0}^{T-t} \gamma^{l} r_{t+l} \nabla_{\theta} \log \pi\left(a_{t} \mid s_{t} ; \theta\right)\right]
$$

## Stochastic approximation

- Full gradient:

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\nabla_{\theta} R^{\pi}=\mathbb{E}_{s_{0: T}, a_{0: T}}\left[\sum_{t=0}^{T} \gamma^{t} \Psi_{t} \nabla_{\theta} \log \pi\left(a_{t} \mid s_{t} ; \theta\right)\right]
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- $\sum_{l=0}^{T-t} \gamma^{l} r_{t+l}-b\left(s_{t}\right)$, where $b\left(s_{t}\right)$ is a baseline, often $b\left(s_{t}\right)=V^{\pi}\left(s_{t}\right)$


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- $\sum_{l=0}^{T-t} \gamma^{l} r_{t+l}-b\left(s_{t}\right)$, where $b\left(s_{t}\right)$ is a baseline, often $b\left(s_{t}\right)=V^{\pi}\left(s_{t}\right)$
- $Q^{\pi}\left(s_{t}, a_{t}\right)-V^{\pi}\left(s_{t}\right)$-advantage function, usually intractable


## Memory problems

## Markov Decision Process



## Partially-observable Markov Decision Process



## Partially-observable Markov Decision Process



## Partially-observable

 Markov Decision Process

## PO-MDP and Memory

- Trajectory $s_{0}, o_{0}, a_{0}, s_{1}, r_{0}, o_{1}, a_{1}, s_{2}, r_{1}, o_{2} \ldots$
- $s_{t+1} \sim p\left(s_{t+1} \mid s_{t}\right)$
- $o_{t} \sim p\left(o_{t} \mid s_{t}\right)$
- $a_{t} \sim \pi\left(a_{t} \mid o_{t}\right)$
- $r_{t}=r\left(s_{t}, a_{t}, s_{t+1}\right)$


## PO-MDP and Memory

- Trajectory $s_{0}, o_{0}, a_{0}, s_{1}, r_{0}, o_{1}, a_{1}, s_{2}, r_{1}, o_{2} \ldots$
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- $a_{t} \sim \pi\left(a_{t} \mid o_{t}\right)$
- $r_{t}=r\left(s_{t}, a_{t}, s_{t+1}\right)$
- Memory assumption:
- there exists a memory $m_{t}=\operatorname{mem}\left(m_{t-1}, o_{t-1}\right)$
- such that $s_{t} \approx f\left(o_{t}, m_{t}\right)$


## LSTM Agent



## Recurrent policy gradients

- Stochastic gradient estimate:

$$
\begin{aligned}
& \widetilde{\nabla}_{\theta} R^{\pi}=\sum_{t=0}^{T} \gamma^{t} \Psi_{t} \nabla_{\theta} \log \pi\left(a_{t} \mid m_{t} ; \theta_{A}\right), \quad m_{t}=\operatorname{mem}\left(m_{t-1}, o_{t} ; \theta_{M}\right) \\
& \Psi_{t}=\sum_{l=0}^{T-t} \gamma^{l} r_{t+l}
\end{aligned}
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\widetilde{\nabla}_{\theta} R^{\pi}=\sum_{t=0}^{T} \gamma^{t} \Psi_{t} \nabla_{\theta} \log \pi\left(a_{t} \mid m_{t} ; \theta_{A}\right), \quad m_{t}=\operatorname{mem}\left(m_{t-1}, o_{t} ; \theta_{M}\right)
$$

- $\Psi_{t}=\sum_{l=0}^{T-t} \gamma^{l} r_{t+l}$
- Backpropagation through time:

$$
\begin{aligned}
\widetilde{\nabla}_{\theta_{M}} R^{\pi} & =\sum_{t=0}^{T} \gamma^{t} \Psi_{t} \frac{\partial \log \pi\left(a_{t} \mid m_{t} ; \theta_{A}\right)}{\partial m_{t}} G_{t} \\
G_{t} & =\frac{\partial m_{t}}{\partial \theta_{M}}+\frac{\partial m_{t}}{\partial m_{t+1}} G_{t+1}
\end{aligned}
$$

## Will this work out of box?

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## Will this work out of box?



- High variance of gradients
- Usual problems with backpropagation through time
- Exploding / vanishing gradients
- Cannot work in a continuous settings


## Variance reduction

- Stochastic gradient estimate:

$$
\widetilde{\nabla}_{\theta} R^{\pi}=\sum_{t=0}^{T} \gamma^{t} \Psi_{t} \nabla_{\theta} \log \pi\left(a_{t} \mid m_{t} ; \theta\right), \quad m_{t}=\operatorname{mem}\left(m_{t-1}, o_{t} ; \theta\right)
$$

- $\Psi_{t}=\sum_{l=0}^{T-t} \gamma^{l} r_{t+l}$ - easy to compute, high variance
- $\Psi_{t}=\sum_{l=0}^{T-t} \gamma^{l} r_{t+l}-b\left(s_{t}\right)$ - baselined estimate
- The optimal baseline is $\frac{\mathbb{E}\left[\left(\sum_{l=0}^{T-t} \gamma^{l} r_{t+l}\right)\left(\nabla_{\theta_{j}} \log \pi\left(a_{t} \mid m_{t}\right)\right)^{2}\right]}{\mathbb{E}\left[\left(\nabla_{\theta_{j}} \log \pi\left(a_{t} \mid m_{t}\right)\right)^{2}\right]}$
- Another important case: $b\left(s_{t}\right)=V^{\pi}\left(s_{t}\right)$


## Learning the Value function



## Learning the Value function



## Final(?) learning algorithm

Repeat until convergence:

1. Collect trajectory $\left\{\left(o_{t}, a_{t}, r_{t}\right)\right\}_{t=0}^{T}$
2. Update policy parameters using $\widetilde{\nabla}_{\theta} R=\sum_{t=0}^{T} \gamma^{t} \Psi_{t} \nabla_{\theta} \log \pi\left(a_{t} \mid m_{t}^{\pi} ; \theta_{A}\right)$
3. Update recurrent parameters using BPTT
4. Update baseline parameters using $\nabla_{\theta_{V}} \sum_{t=0}^{T}\left(V\left(m_{t}^{V} ; \theta_{V}\right)-\sum_{l=0}^{T} \gamma^{l} r_{t+l}\right)^{2}$

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## Learning LSTM policies

- Gradients wrt recurrent parameters are bad after K steps
- For LSTM K is larger than for RNN, but still a finite number
- Continous setting will require large amount of memory
- An obvious solution is to truncate BPTT after K steps
- This limits the range of learned dependencies
- Gradient estimate:

$$
\widetilde{\nabla}_{\theta} R=\sum_{t=0}^{T} \Psi_{t} \nabla_{\theta} \log \pi\left(a_{t} \mid m_{t}^{\pi} ; \theta_{A}\right)
$$

- Consider our advantage estimator:

$$
\Psi_{t}=\sum_{l=0}^{T} \gamma^{l} r_{t+l}-V^{\pi}\left(s_{t}\right)
$$

## Eligibility traces

Gradient estimate:

$$
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$$

- Let's analyze our advantage estimator

$$
\begin{aligned}
\Psi_{t} & =\sum_{l=0}^{T} \gamma^{l} r_{t+l}-V^{\pi}\left(s_{t}\right) \\
& =\sum_{l=0}^{K} \gamma^{l} r_{t+l}+\sum_{l=K+1}^{T} \gamma^{l} r_{t+l}-V^{\pi}\left(s_{t}\right)
\end{aligned}
$$

## Eligibility traces

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\end{aligned}
$$

- Without changing the expectation of gradient we can use

$$
\Psi_{t}=\sum_{l=0}^{K} \gamma^{l} r_{t+l}+\gamma^{K} \underbrace{V^{\pi}\left(s_{t+K+1}\right)}_{\approx V\left(m_{t+K+1} ; \theta_{V}\right)}-V\left(m_{t}^{V} ; \theta_{V}\right)
$$

## Bootstrapping the Baseline

- New error function for the Baseline network

$$
\sum_{t=0}^{T}\left(\sum_{l=0}^{K} \gamma^{l} r_{t+l}+\gamma^{K} V\left(m_{t+K+1}^{V} ; \theta_{M}\right)-V\left(m_{t}^{V} ; \theta_{M}\right)\right)^{2} \rightarrow \min _{\theta_{V}}
$$

- Memory dynamics is controlled by a second LSTM:

$$
m_{t}^{V}=\operatorname{mem}\left(m_{t-1}^{V}, o_{t} ; \theta_{V}\right)
$$

# Empirical results 

## Latch task

| class + noise |
| :---: |
| noise class query  reward      <br> 1.03 0.97 0.24 -0.92 1.2 1.123 -0.05 $?$  |

## Latch task



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## Latch task

- Sequences of length 100 are already hard to learn


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## Latch task

- Sequences of length 100 are already hard to learn
- Eliglibility traces work sometimes, but not very stable
- Curriculum learning:
- Train on shorter sequences
- Increase sequence length over time
- Works well even with truncated BPTT, but no guarantees


## EAT game

## Fruit 1 Fruit 2 Fruit 3



## EAT game

Fruit $1 \quad$ Fruit $2 \quad$ Fruit 3


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## EAT game



## Eat game

- Given enough time LSTM can learn the optimal strategy
- Variance reduction techniques and advances optimization methods dramatically improve convergence


## Big episode EAT

Fruit 1 Fruit 2 Fruit 3


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- Couldn't approximate the Value function well
- The number of contexts is the main bottleneck
- Cannot be handled by curriculum learning directly
- Dirty trick with setting $\Psi_{t}=r_{t}$ worked
- Since our strategy is recurrent future rewards influence gradients at time t
- Prone to bad value function

