Deep Reinforcement Learning with Memory

SERGEY BARTUNOV, HSE, MOSCOW

RL basics



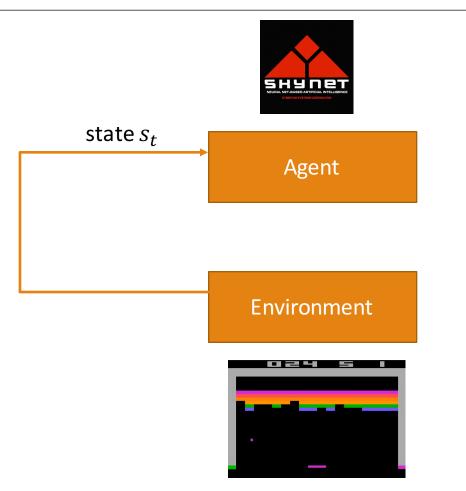
Agent

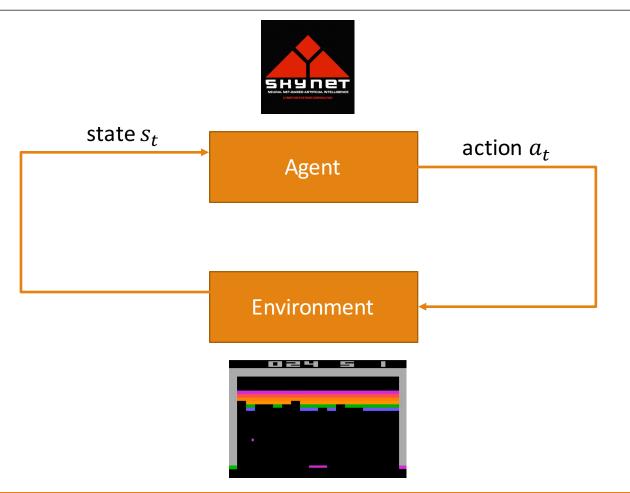


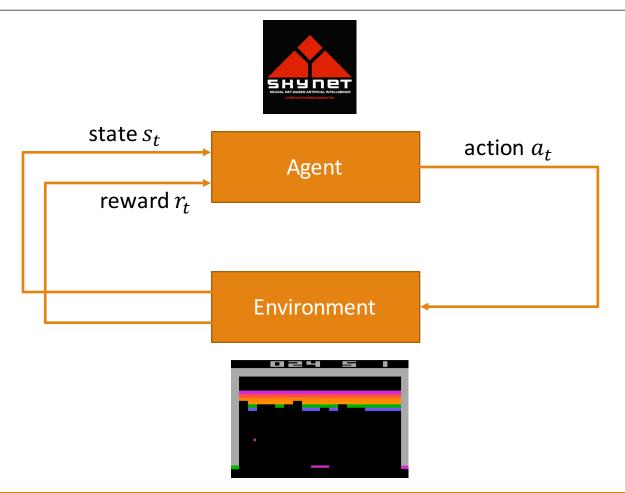


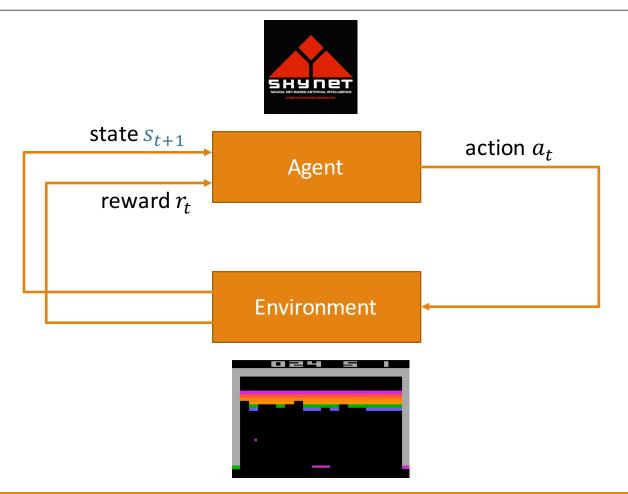
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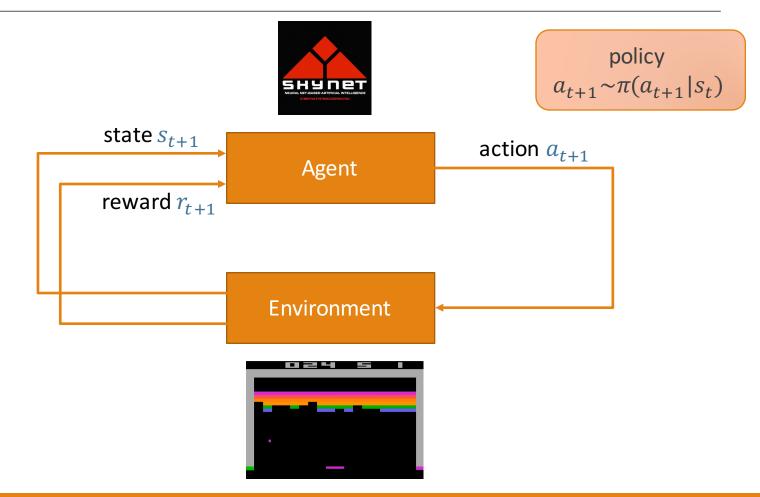


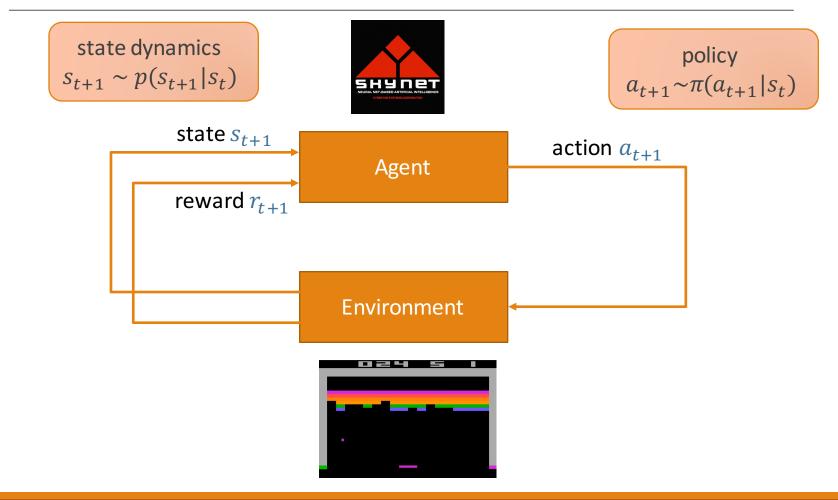


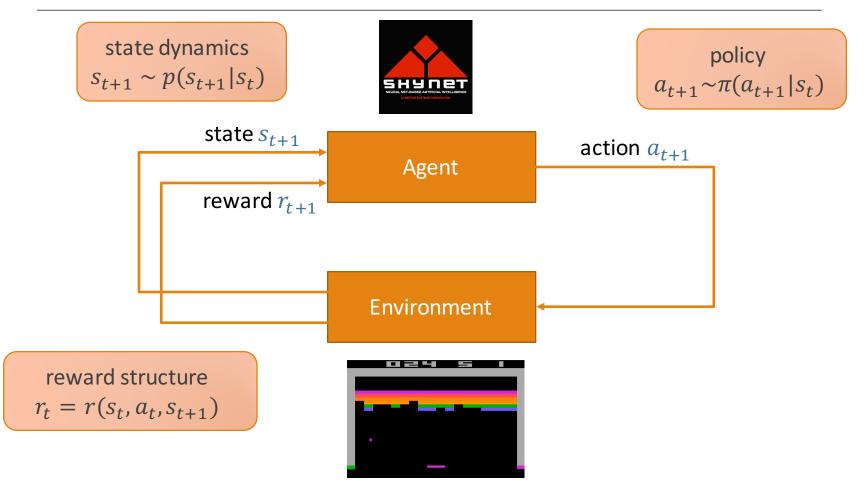












• Trajectory $s_0, a_0, s_1, r_0, a_1, s_2, r_1, \dots$

- $s_{t+1} \sim p(s_{t+1}|s_t)$
- $a_t \sim \pi(a_t | s_t)$
- $\bullet r_t = r(s_t, a_t, s_{t+1})$

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- Action-state function: $Q^{\pi}(s_t, a_t) = \mathbb{E}_{s_{t:T}, a_{t+1:T}} [\sum_{l=0}^{T} \gamma^l r_{t+l}]$ • $V^{\pi}(s_t) = \mathbb{E}_{a_t} Q^{\pi}(s_t, a_t)$

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Policy gradient can be written as follows:

$$\nabla_{\theta} R^{\pi} = \mathbb{E}_{s_{0:T}, a_{0:T}} \left[\sum_{t=0}^{T} \gamma^{t} \sum_{l=0}^{T-t} \gamma^{l} r_{t+l} \nabla_{\theta} \log \pi(a_{t} | s_{t}; \theta) \right]$$

• Full gradient:

$$\nabla_{\theta} R^{\pi} = \mathbb{E}_{s_{0:T}, a_{0:T}} \left[\sum_{t=0}^{T} \gamma^{t} \Psi_{t} \nabla_{\theta} \log \pi(a_{t} | s_{t}; \theta) \right]$$

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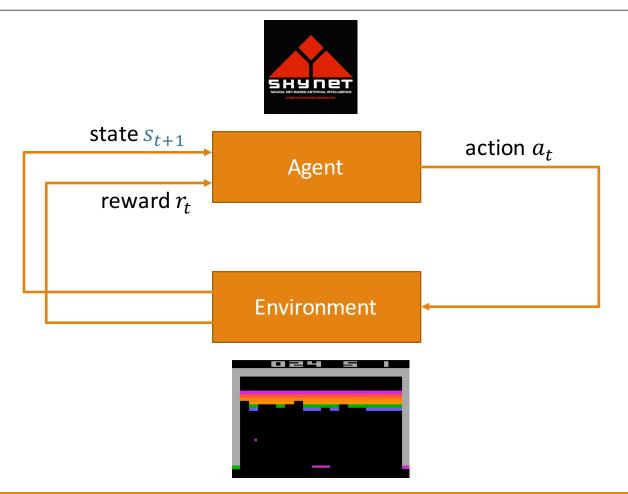
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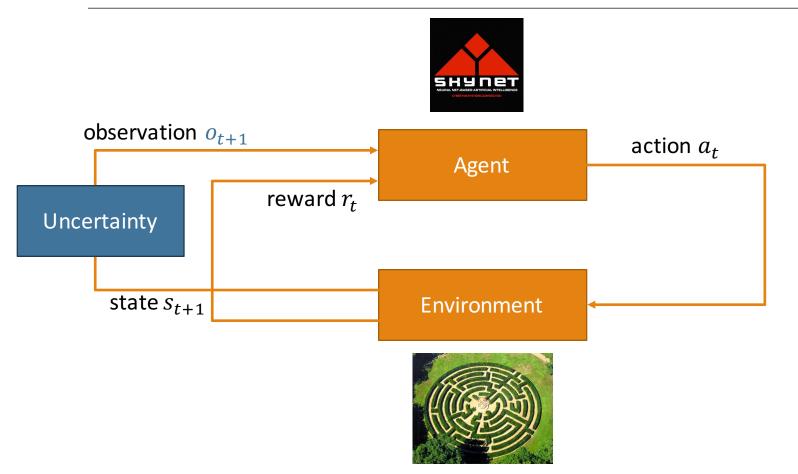
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 - $Q^{\pi}(s_t, a_t) V^{\pi}(s_t)$ advantage function, usually intractable

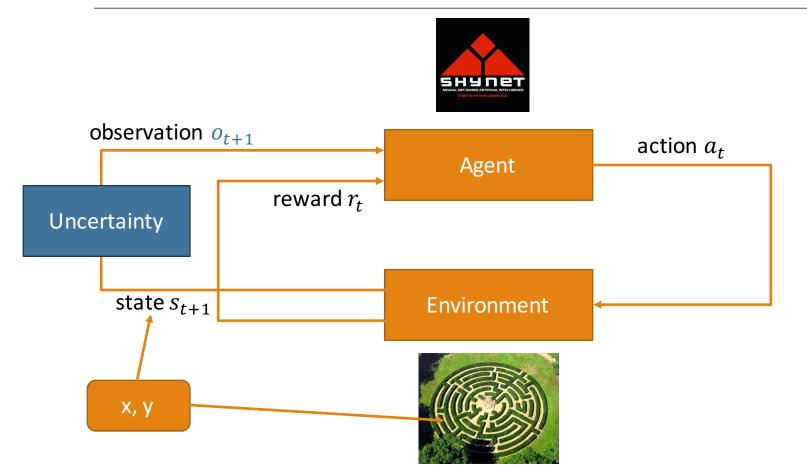
Memory problems



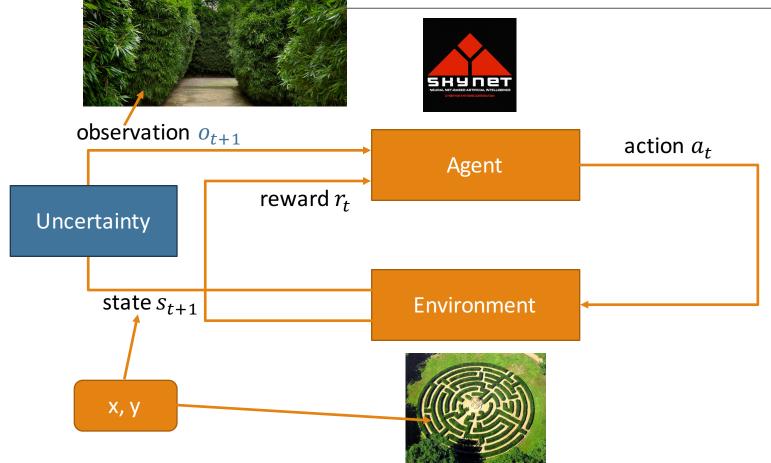
Partially-observable Markov Decision Process



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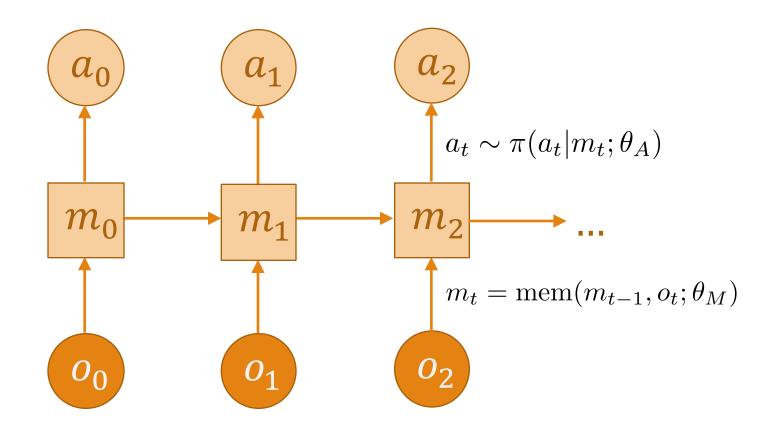
PO-MDP and Memory

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 - $a_t \sim \pi(a_t | o_t)$
 - $r_t = r(s_t, a_t, s_{t+1})$
- Memory assumption:
 - there exists a memory $m_t = \operatorname{mem}(m_{t-1}, o_{t-1})$
 - such that $s_t \approx f(o_t, m_t)$

LSTM Agent



Recurrent policy gradients

Stochastic gradient estimate:

$$\widetilde{\nabla}_{\theta} R^{\pi} = \sum_{t=0}^{T} \gamma^{t} \Psi_{t} \nabla_{\theta} \log \pi(a_{t} | m_{t}; \theta_{A}), \quad m_{t} = \operatorname{mem}(m_{t-1}, o_{t}; \theta_{M})$$

•
$$\Psi_t = \sum_{l=0}^{T-t} \gamma^l r_{t+l}$$

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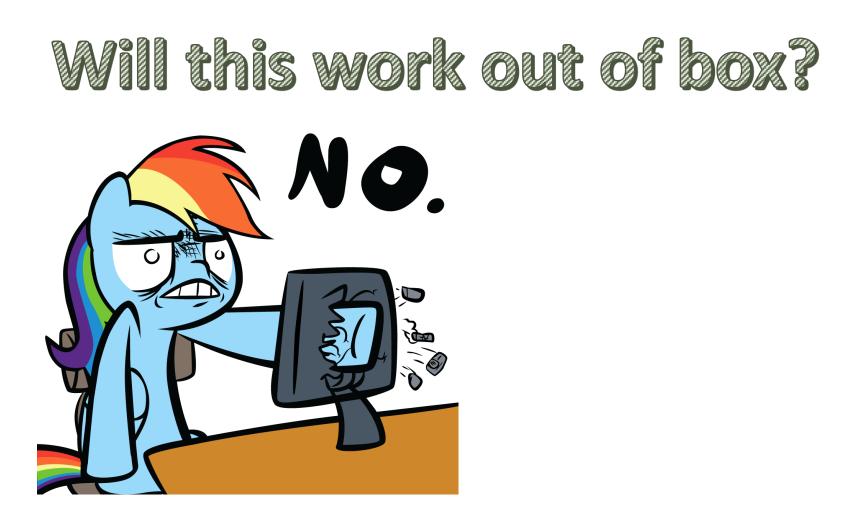
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Backpropagation through time:

$$\widetilde{\nabla}_{\theta_M} R^{\pi} = \sum_{t=0}^{T} \gamma^t \Psi_t \frac{\partial \log \pi(a_t | m_t; \theta_A)}{\partial m_t} G_t$$
$$G_t = \frac{\partial m_t}{\partial \theta_M} + \frac{\partial m_t}{\partial m_{t+1}} G_{t+1}$$

Will this work out of box?



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- High variance of gradients
- Usual problems with backpropagation through time
 - Exploding / vanishing gradients
 - Cannot work in a continuous settings

Variance reduction

Stochastic gradient estimate:

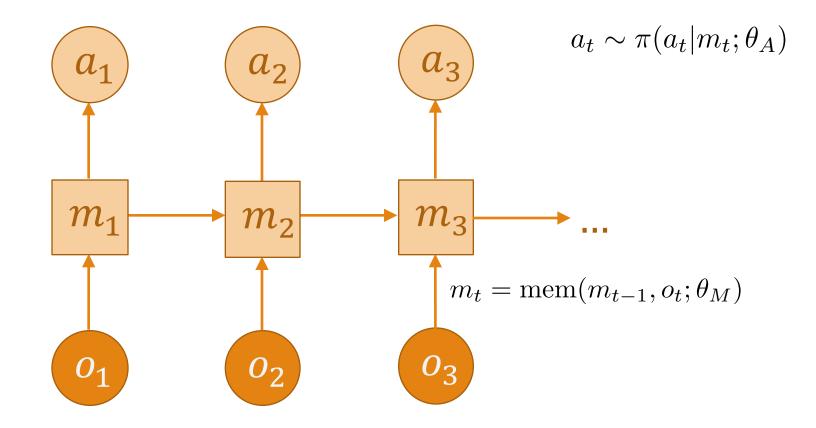
$$\widetilde{\nabla}_{\theta} R^{\pi} = \sum_{t=0}^{T} \gamma^{t} \Psi_{t} \nabla_{\theta} \log \pi(a_{t} | m_{t}; \theta), \quad m_{t} = \operatorname{mem}(m_{t-1}, o_{t}; \theta)$$

•
$$\Psi_t = \sum_{l=0}^{T-t} \gamma^l r_{t+l}$$
 - easy to compute, high variance
• $\Psi_t = \sum_{l=0}^{T-t} \gamma^l r_{t+l} - b(s_t)$ - baselined estimate

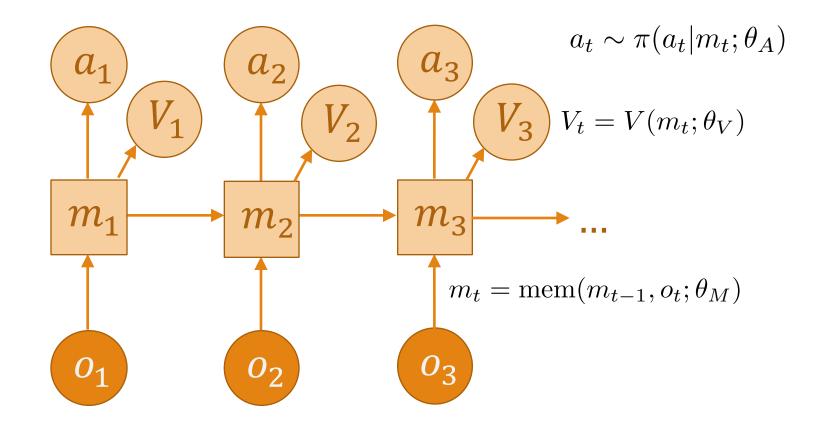
• The optimal baseline is $\frac{\mathbb{E}[(\sum_{l=0}^{T-t} \gamma^l r_{t+l})(\nabla_{\theta_j} \log \pi(a_t | m_t))^2]}{\mathbb{E}[(\nabla_{\theta_j} \log \pi(a_t | m_t))^2]}$

• Another important case: $b(s_t) = V^{\pi}(s_t)$

Learning the Value function



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Final(?) learning algorithm

Repeat until convergence:

- 1. Collect trajectory $\{(o_t, a_t, r_t)\}_{t=0}^T$
- 2. Update policy parameters using $\widetilde{\nabla}_{\theta} R = \sum_{t=0}^{T} \gamma^{t} \Psi_{t} \nabla_{\theta} \log \pi(a_{t} | m_{t}^{\pi}; \theta_{A})$
- 3. Update recurrent parameters using BPTT
- 4. Update baseline parameters using $\nabla_{\theta_V} \sum_{t=0}^T (V(m_t^V; \theta_V) \sum_{l=0}^T \gamma^l r_{t+l})^2$

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Actual (wrong) objective:

$$\mathbb{E}\left[\sum_{t=0}^{T} \gamma^{t} r_{t}\right] - \mathbb{E}\left[\sum_{t=0}^{T} \sum_{l=0}^{T-t} (r_{t+l} - V(m_{t}; \theta_{V}))^{2}\right]$$

Learning LSTM policies

- Gradients wrt recurrent parameters are bad after K steps
 - For LSTM K is larger than for RNN, but still a finite number
- Continuous setting will require large amount of memory
- An obvious solution is to truncate BPTT after K steps
 - This limits the range of learned dependencies
- Gradient estimate:

$$\widetilde{\nabla}_{\theta} R = \sum_{t=0}^{T} \Psi_t \nabla_{\theta} \log \pi(a_t | m_t^{\pi}; \theta_A)$$

Consider our advantage estimator:

$$\Psi_t = \sum_{l=0}^T \gamma^l r_{t+l} - V^{\pi}(s_t)$$

Eligibility traces

•Gradient estimate:

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Let's analyze our advantage estimator

$$\Psi_{t} = \sum_{l=0}^{T} \gamma^{l} r_{t+l} - V^{\pi}(s_{t})$$
$$= \sum_{l=0}^{K} \gamma^{l} r_{t+l} + \sum_{l=K+1}^{T} \gamma^{l} r_{t+l} - V^{\pi}(s_{t})$$

Eligibility traces

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$$= \sum_{l=0}^{K} \gamma^{l} r_{t+l} + \sum_{l=K+1}^{T} \gamma^{l} r_{t+l} - V^{\pi}(s_{t})$$

Without changing the expectation of gradient we can use

$$\Psi_t = \sum_{l=0}^K \gamma^l r_{t+l} + \gamma^K \underbrace{V^{\pi}(s_{t+K+1})}_{\approx V(m_{t+K+1}^V;\theta_V)} - V(m_t^V;\theta_V)$$

Bootstrapping the Baseline

New error function for the Baseline network

$$\sum_{t=0}^{T} \left(\sum_{l=0}^{K} \gamma^{l} r_{t+l} + \gamma^{K} V(m_{t+K+1}^{V}; \theta_{M}) - V(m_{t}^{V}; \theta_{M}) \right)^{2} \to \min_{\theta_{V}}$$

Memory dynamics is controlled by a second LSTM:

$$m_t^V = \operatorname{mem}(m_{t-1}^V, o_t; \theta_V)$$

Empirical results

class + noise				noise			class query	reward
1.03	0.97	0.24	-0.92	1.2	1.123	-0.05	?	

time

class + noise			noise				class query	reward
1.03	0.97	0.24	-0.92	1.2	1.123	-0.05	?	
							class = +1	reward = +1

time

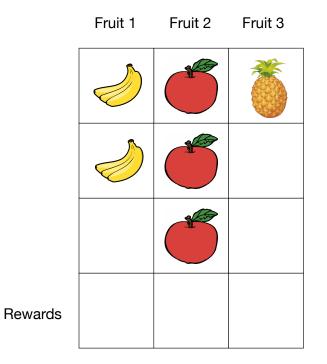
class + noise			noise				class query	reward
1.03	0.97	0.24	-0.92	1.2	1.123	-0.05	?	
							class = -1	reward = -1

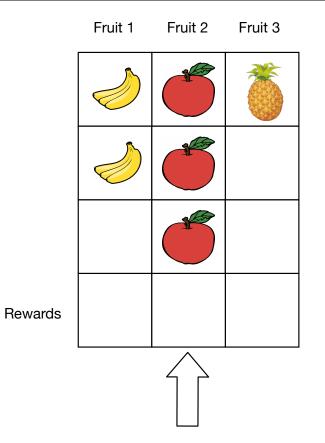
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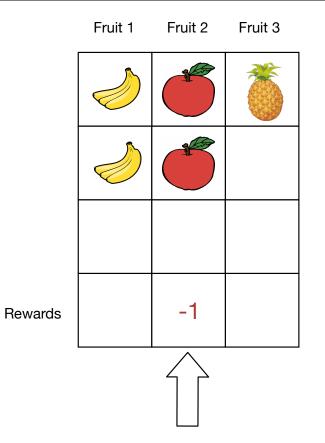
Sequences of length 100 are already hard to learn

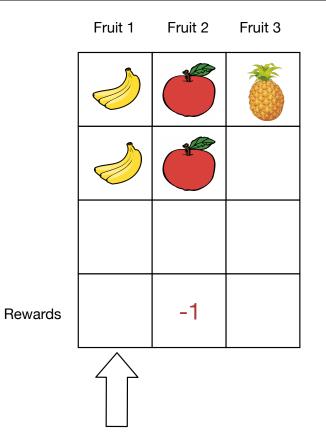
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- Eliglibility traces work sometimes, but not very stable

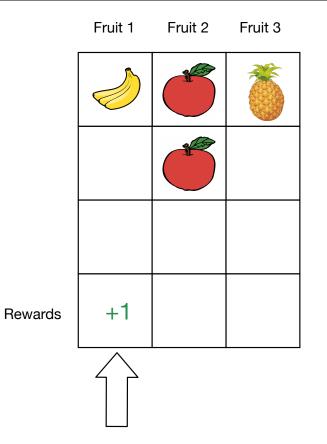
- Sequences of length 100 are already hard to learn
- Eliglibility traces work sometimes, but not very stable
- Curriculum learning:
 - Train on shorter sequences
 - Increase sequence length over time
 - Works well even with truncated BPTT, but no guarantees

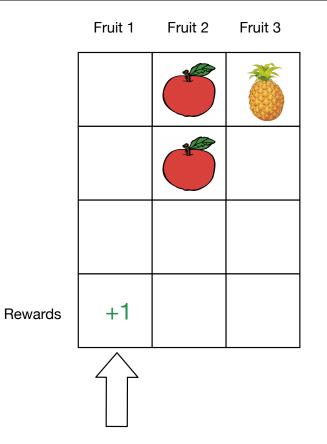


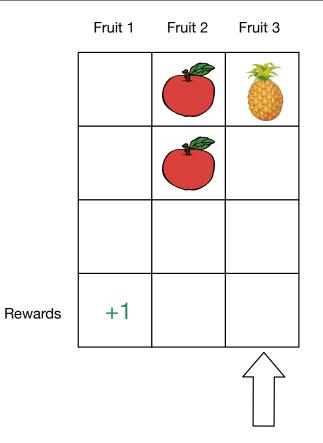


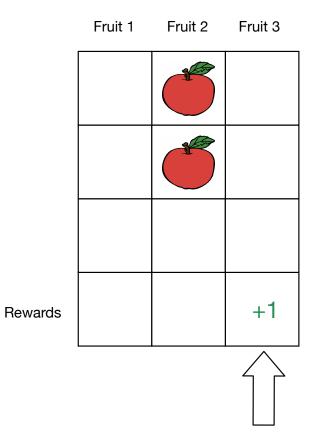


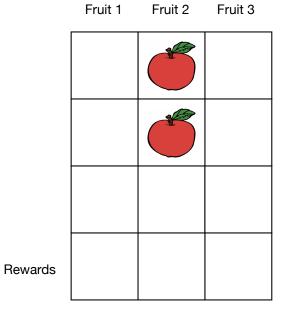








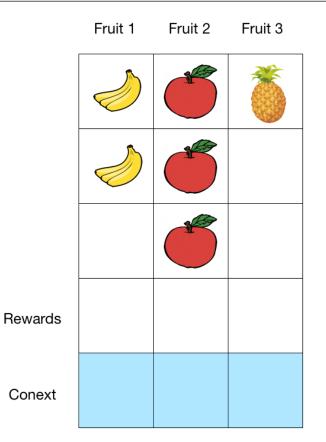


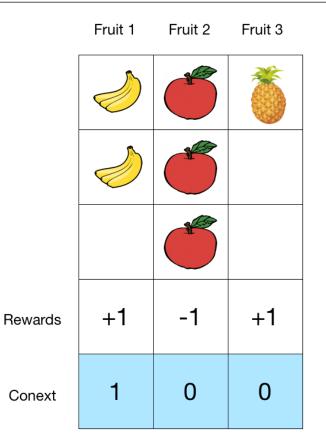


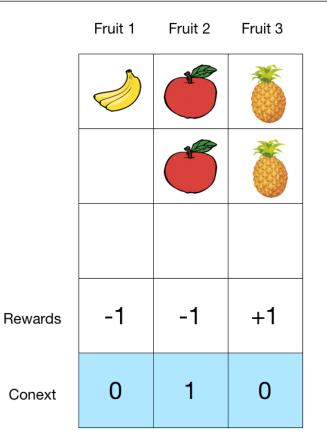
PASS

Eat game

- Given enough time LSTM can learn the optimal strategy
- Variance reduction techniques and advances optimization methods dramatically improve convergence







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- Dirty trick with setting $\Psi_t = r_t$ worked
 - Since our strategy is recurrent future rewards influence gradients at time t
 - Prone to bad value function