# Aggregation of data from different sources in traffic flow tasks

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Research goal Example of initialisation

## Motivation

#### Motivation

Traffic flow mathematical models require accurate data for its initialisation and solving.

Problems with traffic data:

- Traffic detectors data are accurate, but do not cover all considered parts of transport network
- GPS-track data has low accuracy, but covers all considered parts of transport network

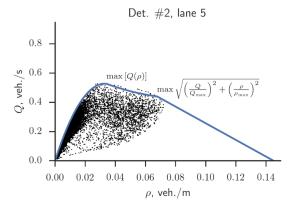
Considered environments:

- highway itself
- highway entrances and exits

#### Introduction

Highway Entrances and exits Summary Research goal Example of initialisation

## Example of initialisation



#### Figure: Fundamental diagram for Moscow Ring Road segment

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Problem statement Speed transformation Experiments

## Main assumption

Let  $N_{\text{track},i} \in \mathbb{N}$ ,  $V_{\text{track},i} \in \mathbb{R}_+$  be a number and speed of vehicles extracted from GPS-tracks at moment *i*. Denote by  $N_{\text{est},i} \in \mathbb{R}$  estimation of the real number of vehicles for the moment of time *i*, which is detected by traffic detectors.

#### Main assumption

$$N_{\text{est},i} = f(\mathbf{a}|N_{\text{track},i}, V_{\text{track},i}),$$

where  $f : \mathbb{R}^m \times \mathbb{N} \times \mathbb{R}_+ \to \mathbb{R}$ ,  $\mathbf{a} \in \mathbb{R}^m$  — parameters vector.

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Problem statement Speed transformation Experiments

#### Problem statement

Let  $N_{det} \in \mathbb{N}$  be a number of vehicles detected by traffic detectors, which considered as true number of vehicles.

Optimization problem

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(f(\mathbf{a}|N_{\mathsf{track},i},V_{\mathsf{track},i})-N_{\mathsf{det},i})^2} \to \min_{\mathbf{a}},$$

where n is a number of two-minutes gaps in a chosen time interval.

Function f representation is dependent on data and is discussed below.

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## Speed transformation

Denote by  $V_{\text{est},i} \in \mathbb{R}$  estimation of the real average speed of vehicles for the moment of time *i*, which is detected by traffic detectors.

#### Speed transformation

$$V_{{\rm est},i} = b_1 + b_2 V_{{\rm track},i},$$

where  $b_1$  and  $b_2$  is a solution of the following problem:

$$\sqrt{rac{1}{n}\sum_{i=1}^{n}(b_1+b_2V_{ ext{track},i}-V_{ ext{det},i})^2}
ightarrow \min_{b_1,b_2},$$

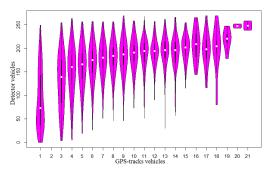
where  $V_{\det,i} \in \mathbb{R}_+$  is a average speed of vehicles detected by traffic detectors.

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## Function *f* representation

Plot dependence  $N_{det}$  vs.  $N_{track}$  and observe dependence similar to log function.



Therefore,

$$f(\mathbf{a}|N_{\text{track},i}, V_{\text{est},i}) = a_0 + a_1 N_{\text{track},i} + a_2 \log (N_{\text{track},i}) + a_3 V_{\text{est},i} + a_4 N_{\text{track},i} / V_{\text{est},i}$$

Problem statement Speed transformation Experiments

## Gain from speed transformation

	$V_{\rm est}$	$V_{\rm track}$
Mean squared error	0.03	0.042
Pearson correlation	0.787	0.672

 $V_{\rm act} = 12.34 + 0.639 V_{\rm track}$ 

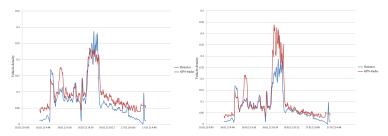


Figure: Plot with vehicle density calculated with (left) and without (right) speed transformation.

Problem statement Speed transformation Experiments

### Parameter optimization

For vehicle density higher than 0.05:

$$N_{
m est} = 157.78 + 4.54 N_{
m track} - 4.59 \log(N_{
m track}) + 0.153 V_{
m est} - -85.069 N_{
m track} / V_{
m track}.$$

For vehicle density less than 0.05:

$$\begin{split} \textit{N}_{\rm est} &= 117.75 + 2.11\textit{N}_{\rm track} + 41.55 \log(\textit{N}_{\rm track}) - 0.327\textit{V}_{\rm est} - \\ &- 128.89\textit{N}_{\rm track} / \textit{V}_{\rm est} \end{split}$$

P-values for all items  $\leq 10^{-5}$  and therefore every item is significant.

	Train	Test <sub>1</sub>	Test <sub>2</sub>	Test <sub>3</sub>	Test <sub>4</sub>
Mean squared error	0.03	0.0363	0.0382	0.0339	0.0393
Pearson correlation	0.787	0.823	0.80	0.85	0.65

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## Vehicle density estimation on train data

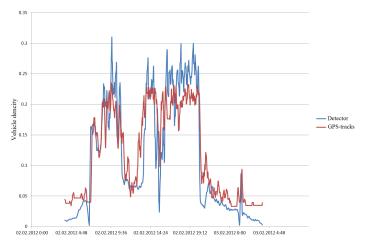
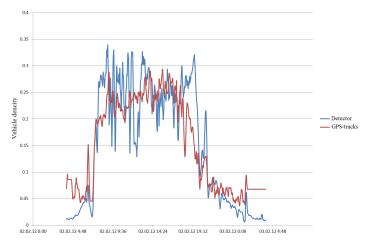


Figure: Vehicle density averaged on 10-minutes obtained after train and ground truth.

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## Vehicle density estimation on test data



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## Quality of model for on-line prediction

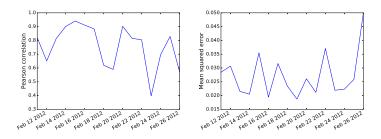


Figure: Correlation (left) and mean squared error (right) averaged on 10-minutes obtained after 7-days learning experiment.

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Problem statement Experiments

## Entrances and exits properties

#### Specific issues for entrances and exits:

- extremely small amount of data
- data from traffic detectors is not ground truth

Let  $N_{ain} \in \mathbb{R}_+$ ,  $N_{aout} \in \mathbb{R}_+$  be in and out vehicles estimation in highway crossroad.

Denote by  $N_{in} \in \mathbb{R}$  total amount of vehicles entered the highway and  $N_{out} \in \mathbb{R}$  total amount of vehicles leave the highway.

#### Balance equation

$$N_{\rm ain} + N_{\rm in} = N_{\rm aout} + N_{\rm out}$$

Computation of  $N_{in}$  and  $N_{out}$  is discussed below.

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Problem statement Experiments

#### Entrances partition

Let  $K_{in} = \{1, \dots, K\}$  be a set of entrance indexes. Denote by  $N_{det}^k$  value of  $N_{det}$  on entrance k.

$$N_{\rm in} = \sum_{k \in K_{\rm in}} N_{\rm det}^k$$

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#### Entrances partition

Let  $K_{in} = \{1, ..., K\}$  be a set of entrance indexes. Denote by  $N_{det}^k$  value of  $N_{det}$  on entrance k.

$$N_{\mathsf{in}} = \sum_{k \in \mathcal{K}_{\mathsf{in}}} N_{\mathsf{det}}^k$$

#### Problem

There exists a set  $K_{intrack} \subset K_{in}$  such that  $\forall k \in K_{intrack} N_{det}^k$  is undefined.

Therefore,  $K_{in} = K_{indet} \cup K_{intrack}$ , such that  $K_{intrack} \cap K_{indet} = \emptyset$  and

- for  $k \in K_{intrack}$  we do not know  $N_{det}^k$
- for  $k \in K_{indet}$  we do know  $N_{det}^k$

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Problem statement Experiments

## $N_{\rm in}$ computation

#### Assumption

For 
$$k \in K_{\text{intrack}} \ N_{\text{det},i}^k = f(\mathbf{a} | N_{\text{track},i}^k, V_{\text{est},i}^k)$$

Denote by  $I_{in}^k$  a set of time indexes *i* such that we have both  $N_{det,i}^k$  and  $N_{track,i}^k$  data for *k*-th entrance.

#### Optimization problem

$$\sqrt{\frac{1}{|I_{\text{in}}^{k^*}|}\sum_{i\in I_{\text{in}}^{k^*}} (f(\mathbf{a}|N_{\text{track},i}^{k^*}, V_{\text{est},i}^{k^*}) - N_{\text{det},i}^{k^*})^2} \to \min_{\mathbf{a}},$$

where  $N_{\text{track},i}^{k*}$ ,  $V_{\text{track},i}^{k*}$ ,  $N_{\text{det},i}^{k*}$  is  $N_{\text{track}}$ ,  $V_{\text{track}}$ ,  $N_{\text{det}}$  for entrance  $k^* \in K_{\text{indet}}$ , which has the large amount of GPS-track data in the *i*-th moment of time,  $i \in I_{\text{in}}^{k^*}$ .

Problem statement

## Problem statement for entrances and exits

Let  $N_{\text{estin}}$ ,  $N_{\text{estout}}$  be estimation of  $N_{\text{in}}$ ,  $N_{\text{out}}$ . Then to find them we propose to solve the following optimization problem

Optimization problem

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$$(N_{ain} + N_{estin} - N_{aout} - N_{estout})^2 \rightarrow \min_{\substack{N_{estin}, N_{estout}}} \\ s.t. \sum_{i \in I_{in}} |N_{estin,i} - N_{in,i}| + \sum_{i' \in I_{out}} |N_{estout,i'} - N_{out,i'}| < \delta, \\ where I_{in} = \bigcap_{\substack{k \in \mathcal{K}_{intrack}}} I_{in}^k, I_{out} = \bigcap_{\substack{k \in \mathcal{K}_{outtrack}}} I_{out}^k \text{ and } \delta \text{ is appropriate} \\ approximation error.} \end{cases}$$

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Problem statement Experiments

## Entrances and exits data recovery algorithm

- Choose crossroad, segments related to entrances, exits and segments from which we take data about N<sub>ain</sub>, N<sub>aout</sub>.
- Determine entrances and exits from *K*<sub>intrack</sub>, *K*<sub>outtrack</sub>. Available small amount of data we use to determine parameters of random Poisson process for chosen entrances and exits.
- To initialize proposed algorithm we use Poisson process with obtained parameters and data from traffic detectors if they are available. The proposed algorithm targets to satisfy balance equation.

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Problem statement Experiments

#### Data recovery visualization



Figure: Blue line — data from traffic detector for one of the entrances, green dots — summary of data from data detector and GPS-tracks. Red line — recovered total number of entered vehicles  $N_{\text{estin}}$ .

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## Summary

- We propose algorithms for data recovery on highway using GPS-track data and traffic detectors data.
- We visualize given data to represent target function in the most appropriate way.
- We extend algorithm for highway to highway enters and exits.
- We perform computational experiments for every proposed algorithm.

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# Thank you for your attention!

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