# Aggregation of data from different sources in traffic flow tasks 

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## Motivation

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Traffic flow mathematical models require accurate data for its initialisation and solving.

Problems with traffic data:

- Traffic detectors data are accurate, but do not cover all considered parts of transport network
- GPS-track data has low accuracy, but covers all considered parts of transport network
Considered environments:
- highway itself
- highway entrances and exits


## Example of initialisation

Det. \#2, lane 5


Figure: Fundamental diagram for Moscow Ring Road segment

## Main assumption

Let $N_{\text {track }, i} \in \mathbb{N}, V_{\text {track, } i} \in \mathbb{R}_{+}$be a number and speed of vehicles extracted from GPS-tracks at moment $i$.
Denote by $N_{\text {est }, i} \in \mathbb{R}$ estimation of the real number of vehicles for the moment of time $i$, which is detected by traffic detectors.

## Main assumption

$$
N_{\text {est }, i}=f\left(\mathbf{a} \mid N_{\text {track }, i}, V_{\text {track }, i}\right),
$$

where $f: \mathbb{R}^{m} \times \mathbb{N} \times \mathbb{R}_{+} \rightarrow \mathbb{R}, \mathbf{a} \in \mathbb{R}^{m}$ - parameters vector.

## Problem statement

Let $N_{\text {det }} \in \mathbb{N}$ be a number of vehicles detected by traffic detectors, which considered as true number of vehicles.

## Optimization problem

$$
\sqrt{\frac{1}{n} \sum_{i=1}^{n}\left(f\left(\mathbf{a} \mid N_{\text {track }, i}, V_{\text {track }, i}\right)-N_{\text {det }, i}\right)^{2}} \rightarrow \min _{\mathbf{a}}
$$

where $n$ is a number of two-minutes gaps in a chosen time interval.
Function $f$ representation is dependent on data and is discussed below.

## Speed transformation

Denote by $V_{\text {est }, i} \in \mathbb{R}$ estimation of the real average speed of vehicles for the moment of time $i$, which is detected by traffic detectors.

## Speed transformation

$$
V_{\text {est }, i}=b_{1}+b_{2} V_{\text {track }, i},
$$

where $b_{1}$ and $b_{2}$ is a solution of the following problem:

$$
\sqrt{\frac{1}{n} \sum_{i=1}^{n}\left(b_{1}+b_{2} V_{\mathrm{track}, i}-V_{\mathrm{det}, i}\right)^{2} \rightarrow \min _{b_{1}, b_{2}}, ., ~}
$$

where $V_{\text {det }, i} \in \mathbb{R}_{+}$is a average speed of vehicles detected by traffic detectors.

## Function $f$ representation

Plot dependence $N_{\text {det }}$ vs. $N_{\text {track }}$ and observe dependence similar to log function.


Therefore,

$$
\begin{aligned}
& f\left(\mathbf{a} \mid N_{\text {track }, i}, V_{\text {est }, i}\right)=a_{0}+a_{1} N_{\text {track }, i}+a_{2} \log \left(N_{\text {track }, i}\right)+ \\
& +a_{3} V_{\text {est }, i}+a_{4} N_{\text {track }, i} / V_{\text {est }, i}
\end{aligned}
$$

## Gain from speed transformation

$$
V_{\text {est }}=12.34+0.639 V_{\text {track }}
$$

|  | $V_{\text {est }}$ | $V_{\text {track }}$ |
| :---: | :---: | :---: |
| Mean squared error | $\mathbf{0 . 0 3}$ | 0.042 |
| Pearson correlation | $\mathbf{0 . 7 8 7}$ | 0.672 |




Figure: Plot with vehicle density calculated with (left) and without (right) speed transformation.

## Parameter optimization

For vehicle density higher than 0.05 :

$$
\begin{array}{r}
N_{\text {est }}=157.78+4.54 N_{\text {track }}-4.59 \log \left(N_{\text {track }}\right)+0.153 V_{\text {est }}- \\
-85.069 N_{\text {track }} / V_{\text {track }} .
\end{array}
$$

For vehicle density less than 0.05 :

$$
\begin{array}{r}
N_{\text {est }}=117.75+2.11 N_{\text {track }}+41.55 \log \left(N_{\text {track }}\right)-0.327 V_{\text {est }}- \\
\\
-128.89 N_{\text {track }} / V_{\text {est }}
\end{array}
$$

$P$-values for all items $\leq 10^{-5}$ and therefore every item is significant.

|  | Train | Test $_{1}$ | Test $_{2}$ | Test $_{3}$ | Test $_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mean squared error | $\mathbf{0 . 0 3}$ | 0.0363 | 0.0382 | 0.0339 | 0.0393 |
| Pearson correlation | 0.787 | 0.823 | 0.80 | $\mathbf{0 . 8 5}$ | 0.65 |

## Vehicle density estimation on train data



Figure: Vehicle density averaged on 10-minutes obtained after train and ground truth.

## Vehicle density estimation on test data



Figure: Vehicle density averaged on 10-minutes obtained after test ${ }_{1}$ and ground truth.

## Quality of model for on-line prediction




Figure: Correlation (left) and mean squared error (right) averaged on 10-minutes obtained after 7-days learning experiment.

## Entrances and exits properties

## Specific issues for entrances and exits:

- extremely small amount of data
- data from traffic detectors is not ground truth

Let $N_{\text {ain }} \in \mathbb{R}_{+}, N_{\text {aout }} \in \mathbb{R}_{+}$be in and out vehicles estimation in highway crossroad.
Denote by $N_{\text {in }} \in \mathbb{R}$ total amount of vehicles entered the highway and $N_{\text {out }} \in \mathbb{R}$ total amount of vehicles leave the highway.
Balance equation

$$
N_{\text {ain }}+N_{\text {in }}=N_{\text {aout }}+N_{\text {out }}
$$

Computation of $N_{\text {in }}$ and $N_{\text {out }}$ is discussed below.

## Entrances partition

Let $K_{\text {in }}=\{1, \ldots, K\}$ be a set of entrance indexes.
Denote by $N_{\text {det }}^{k}$ value of $N_{\text {det }}$ on entrance $k$.

$$
N_{\mathrm{in}}=\sum_{k \in K_{\text {in }}} N_{\mathrm{det}}^{k}
$$

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$$

## Problem

There exists a set $K_{\text {intrack }} \subset K_{\text {in }}$ such that $\forall k \in K_{\text {intrack }} N_{\text {det }}^{k}$ is undefined.

Therefore, $K_{\text {in }}=K_{\text {indet }} \cup K_{\text {intrack }}$, such that $K_{\text {intrack }} \cap K_{\text {indet }}=\varnothing$ and

- for $k \in K_{\text {intrack }}$ we do not know $N_{\text {det }}^{k}$
- for $k \in K_{\text {indet }}$ we do know $N_{\text {det }}^{k}$


## $N_{\text {in }}$ computation

## Assumption

For $k \in K_{\text {intrack }} N_{\text {det }, i}^{k}=f\left(\mathbf{a} \mid N_{\text {track }, i}^{k}, V_{\text {est }, i}^{k}\right)$
Denote by $l_{\text {in }}^{k}$ a set of time indexes $i$ such that we have both $N_{\text {det }, i}^{k}$ and $N_{\text {track, }}^{k} i$ data for $k$-th entrance.

## Optimization problem

$$
\sqrt{\frac{1}{\left|l_{\mathrm{in}}^{k^{*}}\right|} \sum_{i \in \ell_{\mathrm{in}}^{k^{*}}}\left(f\left(\mathbf{a} \mid N_{\text {track }, i}^{k^{*}}, V_{\text {est }, i}^{k^{*}}\right)-N_{\text {det }, i}^{k^{*}}\right)^{2}} \rightarrow \min _{\mathbf{a}},
$$

where $N_{\text {track }, i}^{k *}, V_{\text {track }, i}^{k^{*}}, N_{\text {det, } i}^{k^{*}}$ is $N_{\text {track }}, V_{\text {track }}, N_{\text {det }}$ for entrance $k^{*} \in K_{\text {indet }}$, which has the large amount of GPS-track data in the $i$-th moment of time, $i \in l_{\text {in }}^{k^{*}}$.

## Problem statement for entrances and exits

Let $N_{\text {estin }}, N_{\text {estout }}$ be estimation of $N_{\text {in }}, N_{\text {out }}$. Then to find them we propose to solve the following optimization problem

## Optimization problem

$$
\begin{aligned}
& \left(N_{\text {ain }}+N_{\text {estin }}-N_{\text {aout }}-N_{\text {estout }}\right)^{2} \rightarrow \min _{N_{\text {estin }}, N_{\text {estout }}} \\
\text { s.t. } & \sum_{i \in l_{\text {in }}}\left|N_{\text {estin }, i}-N_{\text {in }, i}\right|+\sum_{i^{\prime} \in l_{\text {out }}}\left|N_{\text {estout }, i^{\prime}}-N_{\text {out }, i^{\prime}}\right|<\delta,
\end{aligned}
$$

where $I_{\text {in }}=\bigcap_{k \in K_{\text {intrack }}} l_{\text {in }}^{k}, I_{\text {out }}=\bigcap_{k \in K_{\text {outtrack }}} I_{\text {out }}^{k}$ and $\delta$ is appropriate approximation error.

## Entrances and exits data recovery algorithm

- Choose crossroad, segments related to entrances, exits and segments from which we take data about $N_{\text {ain }}, N_{\text {aout }}$.
- Determine entrances and exits from $K_{\text {intrack }}, K_{\text {outtrack }}$. Available small amount of data we use to determine parameters of random Poisson process for chosen entrances and exits.
- To initialize proposed algorithm we use Poisson process with obtained parameters and data from traffic detectors if they are available. The proposed algorithm targets to satisfy balance equation.


## Data recovery visualization



Figure: Blue line - data from traffic detector for one of the entrances, green dots - summary of data from data detector and GPS-tracks. Red line - recovered total number of entered vehicles $N_{\text {estin }}$.

## Summary

- We propose algorithms for data recovery on highway using GPS-track data and traffic detectors data.
- We visualize given data to represent target function in the most appropriate way.
- We extend algorithm for highway to highway enters and exits.
- We perform computational experiments for every proposed algorithm.


## Thank you for your attention!

