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# IRIS IMAGE SEGMENTATION BY PAIRED GRADIENT METHOD WITH PUPIL BOUNDARY REFINEMENT

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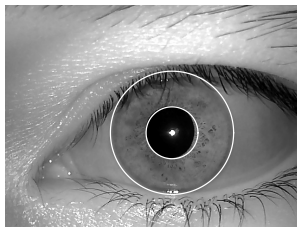
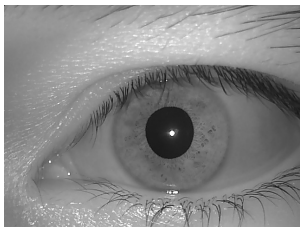
# Problem statement

## Input:

$I$  — grayscale bitmap sized  $W \times H$ . Every pixel is encoded in one byte.

## Output:

An approximation of iris boundaries in an eye image  $I$  by two circles, i.e. to determine center coordinates and the corresponding radii  $(x, y, r)_P$  and  $(x, y, r)_I$ .



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## 1. Daugman's approach

Circular approximation parameters are determined by integro-differential operator:

$$\max_{(r, x_0, y_0)} \left| G_\sigma(r) \frac{\partial}{\partial r} \oint_{(x_0, y_0, r_0)} \frac{I(x, y)}{2\pi r} ds \right|$$

## 2. Wildes' approach and its modifications

Searching for local maxima in the parameter space.

There are modifications, allowing to reduce the computational complexity: gradient-based approaches, randomized algorithms for circle detection, separation of the accumulator parameter space.

## 3. Active contours

## 1. Projection methods

Intensity projection method, gradient projection method, blob detection.

## 2. Morphological methods

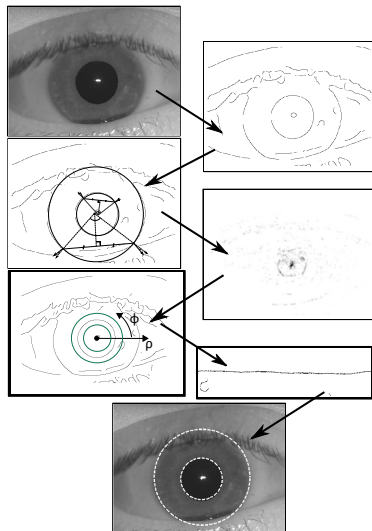
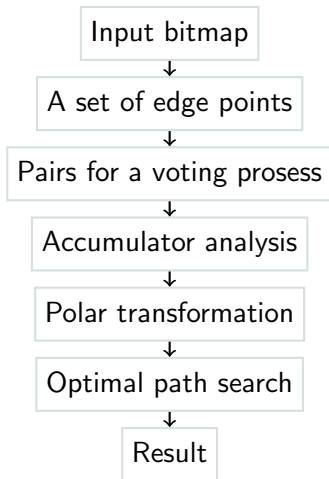
A method of recursive erosion.

## 3. Hough methodology

## 4. Contour-based methods

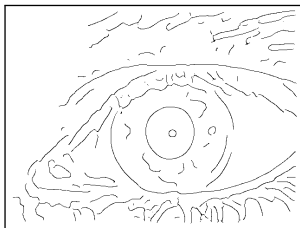
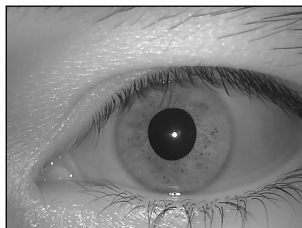
Pupil boundary is considered to be a curve, determined directly by a sequence of pixels and not belonging to any existing class of figures.

# Proposed solution



# Edge points selection

To detect possible edges in an image Canny operator is applied. In the neighborhood of the selected points gradient components  $\mathbf{g}_x(x, y)$  and  $\mathbf{g}_y(x, y)$  are calculated using Sobel masks and then gradient magnitude  $g(x, y)$  and angle  $\phi(x, y)$  are defined. A set of edge points  $G = \{x, y, g(x, y), \phi(x, y)\} = \{\mathbf{L}, \mathbf{W}\}$  is formed.



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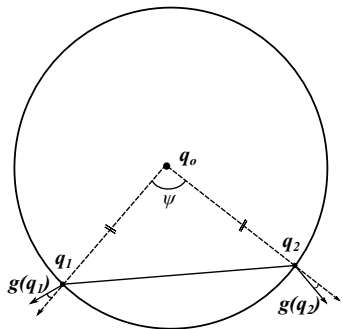
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# Paired Gradient method

## Main concept:



Let  $\mathbf{q} = (x, y)$  be an edge point. Then the selection criteria for a pair  $\{\mathbf{q}_1, \mathbf{q}_2\}$ , corresponding to a hypothetical circle:

$$\|\mathbf{g}(\mathbf{q}_1)\| > T_g,$$

$$\|\mathbf{g}(\mathbf{q}_2)\| > T_g,$$

$$\angle(\mathbf{g}(\mathbf{q}_1), \mathbf{g}(\mathbf{q}_2)) = \psi,$$

$$\|\mathbf{q}_1 - \mathbf{q}_0\| = \|\mathbf{q}_2 - \mathbf{q}_0\|$$

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# Paired Gradient method

If the pair  $\{\mathbf{q}_1, \mathbf{q}_2\}$  is selected, then the parameters  $\mathbf{p}(\mathbf{q}_1, \mathbf{q}_2) = \{x_c, y_c, r\}$  of the corresponding hypothetical circle are calculated as follows:

the coordinates of an interception point  $\mathbf{q}^*$  for the following lines

$$l_1 = \mathbf{q}_1 - t_1 \cdot \mathbf{g}(\mathbf{q}_1),$$

$$l_2 = \mathbf{q}_2 - t_2 \cdot \mathbf{g}(\mathbf{q}_2)$$

specify its center  $(x_c, y_c)$  and the radius can be found as

$$r = \sqrt{(x_1 - x_c)^2 + (y_1 - y_c)^2}.$$

A set of hypothetical circle parameters  $P = \{x_c^i, y_c^i, r^i\}_{i=1}^M$  is formed, where  $M$  is the number of selected pairs.

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## Center search:

The mentioned set  $P = \{x_c^i, y_c^i, r^i\}_{i=1}^M$  is used during the Hough voting process in the accumulator array  $Q$ . The zero-initialized array is filled with the center votes  $\{x_c^i, y_c^i\}$ :

$$Q(x, y) = \sum_{i=1}^M \begin{cases} 1, & \text{if } (x, y) = (x_c^i, y_c^i), \\ 0 & \text{otherwise.} \end{cases}$$

An accumulator element, which received the most votes, i.e. the argument maxima  $\mathbf{q}_1^* = (x_c^*, y_c^*) = \underset{(x,y)}{\operatorname{argmax}} Q(x, y)$  is the most probable center position of the circle, approximating the most expressed iris boundary.

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# Circular approximation

## Center search:

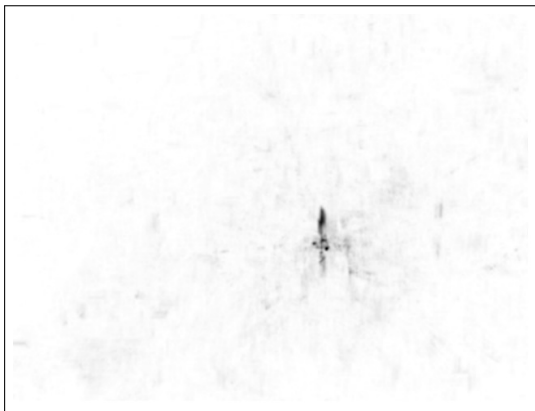


Figure: An accumulator array for  $\psi = \frac{2\pi}{3}$

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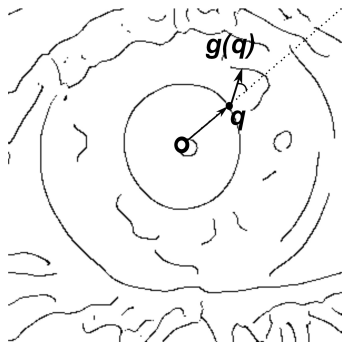
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## Noise suppression:



Considering the found eye center position and using the gradient information, a constraint may be introduced for edge points in  $\mathbf{G}$ :

$$\arccos \left( \frac{\mathbf{q} \cdot \mathbf{g}(\mathbf{q})}{|\mathbf{q}| \cdot |\mathbf{g}(\mathbf{q})|} \right) < T_a$$

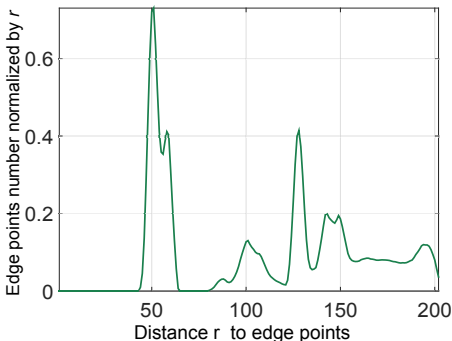
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# Circular approximation

**Radius detection:** To determine the radius a distance histogram  $H(r)$  is built:

$$H(r) = |\{\mathbf{q} : \mathbf{q} = (x, y) \in \mathbf{G}, \|\mathbf{q} - \mathbf{q}_1^*\| \in (r - 0.5, r + 0.5)\}|.$$

Its argument maxima corresponds to the sought-for radius  $r_1^*$ .



**Approximating the second boundary** To detect the second iris boundary limiting constraints are imposed on its inner and outer radii:

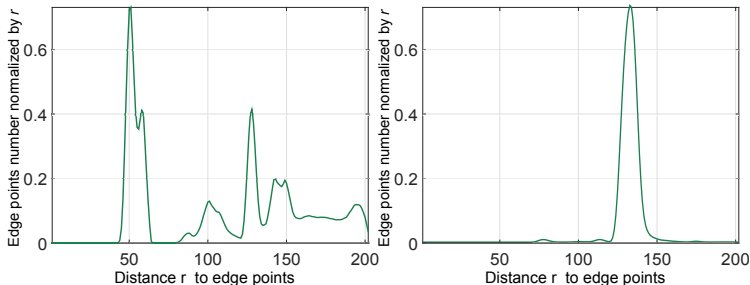
$$r_P > \frac{1}{7}r_1,$$

$$r_P < \frac{3}{4}r_1.$$

$$r_P > \sqrt{(x_I - x_P)^2 + (y_I - y_P)^2}.$$

# Circular approximation

**Approximating the second boundary:** Values of the original histogram in region  $r \in [0; \frac{1}{7}r_1^*] \cup [\frac{3}{4}r_1^*; \frac{4}{3}r_1^*]$ , are set to zero not to detect the already found iris boundary. New argument maxima corresponds to the second sought-for radius  $r_2^*$ .



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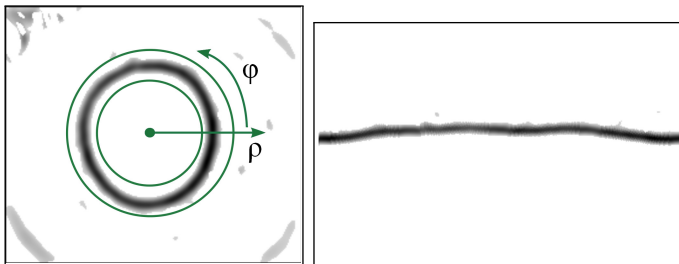
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# Pupil boundary refinement

**Polar representation:** A polar transformation is applied to the edge map with the pole in  $(x_c^*, y_c^*)$ . A narrow zone of the polar representation  $\mathbf{G}_p$  is considered, where  $y \in [r_p - 20; r_p + 20]$ .



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**Circular shortesh path method:** Let there be a contour in the polar representation, defined by a sequence of pixels:

$S = \{\rho(\phi_k)\}_{k=1}^M$ . A cost for the path from  $(n, \rho_n)$  to  $(m, \rho_m)$  consists of two components:

$$C(\rho_n, \rho_m) = C_0(n, \rho_n) + C_1(\rho_n, \rho_m).$$

$$C_0(n, \rho_n) = g(n, \rho_n).$$

$$C_1(\rho_n, \rho_m) = \begin{cases} 0, & \text{if } \rho_n = \rho_m, \\ T_1, & \text{if } |\rho_n - \rho_m| = 1, \\ \infty, & \text{otherwise.} \end{cases}$$

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# Pupil boundary refinement

## Circular shortest path method:

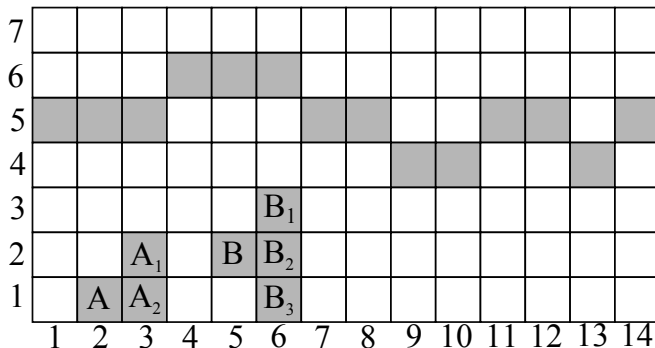


Figure: Neighbour points for a circular path.

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**Circular shortest path method:** For the given path  $S = \{\rho_k\}_{k=1}^{W_p}$  the total cost is calculated:

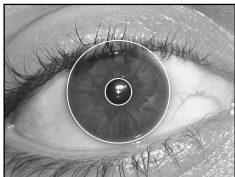
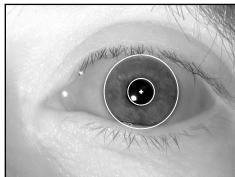
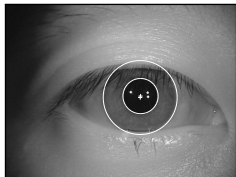
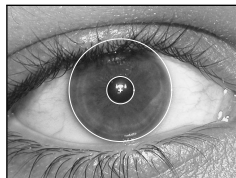
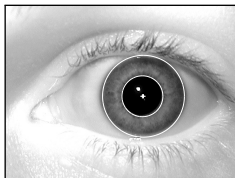
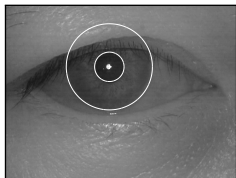
$$C(S) = \sum_{k=1}^{W_p} C(\rho_k, \rho_{k+1}).$$

An optimal contour has the minimal total cost:

$$S^* = \underset{S}{\operatorname{argmin}} C(S).$$

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## Circular approximation:



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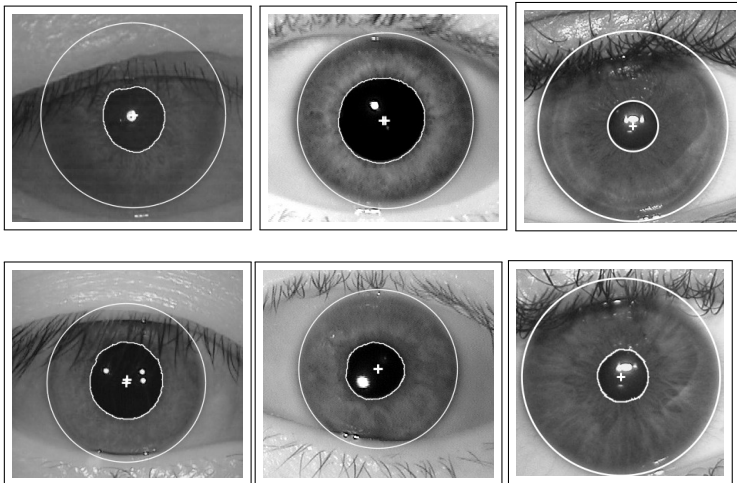
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## Pupil boundary refinement:



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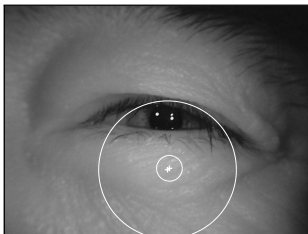
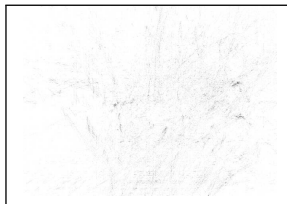
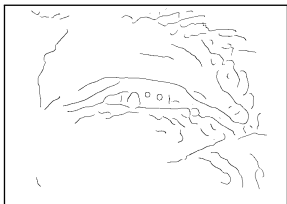
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# Incorrect segmentation

## Narrowed eyelids



## Goals:

- ▶ Testing the iris segmentation system on real data.
- ▶ Building an error plot for further analysis

## Input data format:

Grayscale eye images sized  $640 \times 480$  pixels (CASIA(20000), ND-IRIS(20000), UBI(1207)).

## Quality estimation:

- ▶ Segmentation result:  $\omega = \{x_P, y_P, r_P, x_I, y_I, r_I\}$ .
- ▶ Expert markup:  $\tilde{\omega} = \{\tilde{x}_P, \tilde{y}_P, \tilde{r}_P, \tilde{x}_I, \tilde{y}_I, \tilde{r}_I\}$ .
- ▶ Center detection error:  $S_c(\omega) = \sqrt{(x_P - \tilde{x}_P)^2 + (y_P - \tilde{y}_P)^2} + \sqrt{(x_I - \tilde{x}_I)^2 + (y_I - \tilde{y}_I)^2}$ .
- ▶ Radii estimation error:  $S_r(\omega) = |r_P - \tilde{r}_P| + |r_I - \tilde{r}_I|$ .
- ▶ Total error is the sum:  $S(\omega) = S_c(\omega) + S_r(\omega)$ .
- ▶ Relative errors:  $\varepsilon(\omega) = \frac{S(\omega)}{\tilde{r}_I}, \varepsilon_c(\omega) = \frac{S_c(\omega)}{\tilde{r}_I}$ .
- ▶ Average relative error:  $E = \frac{1}{N} \sum_{i=1}^N \varepsilon(\omega_i)$ .
- ▶ Error distribution histogram:  
 $e(p) = |\{I : \varepsilon(\omega) \leq p\}|, p \in [0; 1]$ .

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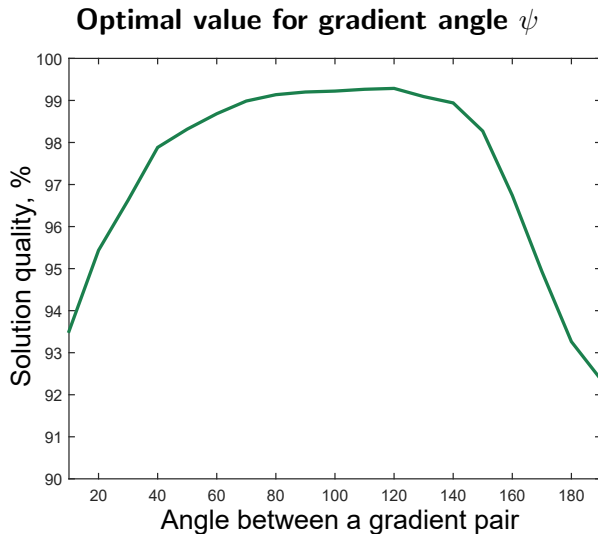
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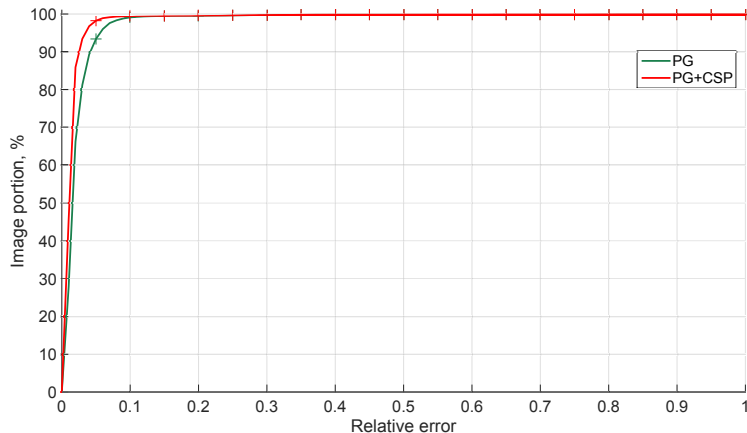
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## A distribution of relative pupil error



Summed relative error  $\varepsilon(\omega)$  distribution, %

$e < 5\%$	$e < 10\%$	$e < 15\%$	$e < 20\%$	$e < 25\%$
32.2	85.33	95.09	98.21	99.02

Center detection relative error  $\varepsilon_c(\omega)$  distribution, %

$e_c < 5\%$	$e_c < 10\%$	$e_c < 15\%$	$e_c < 20\%$	$e_c < 25\%$
73.01	97.03	99.44	99.65	99.78

Average relative error E, %

	Daugman	Ma et al.	Wildes	Masek	PG+CSP
CASIA	1.19	4.79	5.37	5.15	2.51
NDIRIS	1.10	5.92	6.33	5.59	2.24

Average time, ms

$\bar{t}, \text{ms}$	52.31	363.64	379.61	97.52	203.9
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- ▶ A system of methods for detecting iris region in eye image is presented.
- ▶ The system is implemented in C and Matlab.
- ▶ To estimate the overall efficiency, a computational experiment was performed on images from public domain databases.

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# Thank you for your attention!